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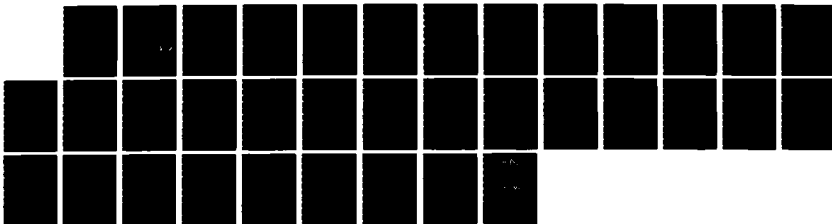
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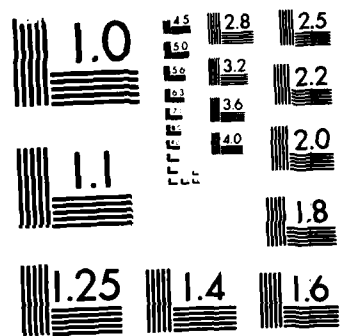
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**Radiant-Energy Penetration Effect  
in the Thermal-Diffusivity Flash Technique  
for Layered and Porous Polymers**

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Materials Sciences Laboratory  
Laboratory Operations  
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El Segundo CA 90245

2 December 1985

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SPACE DIVISION  
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P.O. Box 92960, Worldway Postal Center  
Los Angeles, CA 90009-2960

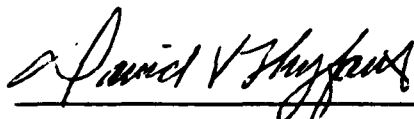
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This report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-85-C-0086 with the Space Division, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009-2960. It was reviewed and approved for The Aerospace Corporation by R. W. Fillers, Director, Materials Sciences Laboratory.

Capt David Thyfault, SD/YNSA, was the project officer for the Mission-Oriented Investigation and Experimentation (MOIE) Program.

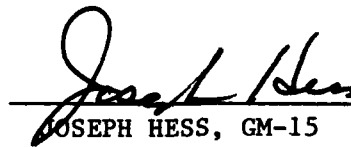
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This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.



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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SD-TR-85-80	2. GOVT ACCESSION NO. AD-A162	3. RECIPIENT'S CATALOG NUMBER 508
4. TITLE (and Subtitle) RADIANT-ENERGY PENETRATION EFFECT IN THE THERMAL-DIFFUSIVITY FLASH TECHNIQUE FOR LAYERED AND POROUS POLYMERS		5. TYPE OF REPORT & PERIOD COVERED
7. AUTHOR(s) Robert S. Bretzlaff		6. PERFORMING ORG. REPORT NUMBER TR-0086(6935-03)-2
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Aerospace Corporation El Segundo, Calif. 90245		8. CONTRACT OR GRANT NUMBER(s) F04701-85-C-0086
11. CONTROLLING OFFICE NAME AND ADDRESS Space Division Los Angeles Air Force Station Los Angeles, Calif. 90009-2960		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE 2 December 1985
		13. NUMBER OF PAGES 33
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Flash technique                      Radiant energy penetration Heat conduction problem              Thermal diffusivity Insulation material Laplace transform solution		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Thermal insulation characterization of polymers and other materials is an important requirement for present and future aerospace missions, as well as for the home construction industry. Some dispersed composites and layered samples have been successfully characterized by the thermal-diffusivity flash technique, while the interpretation of these and other experiments for systems remains problematical. One refinement for layered-sample data reduction is investigated here. An exponentially decaying spatial		

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19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

penetration of radiant energy into the sample is accounted for. Under the conditions chosen for numerical evaluation, an 18% reduction in the back-face rise time,  $t_{1/2}$ , is predicted for a case in which on 5% as much radiant energy is deposited on the midplane as on the front surface. It is suggested how the use of the entire  $V(t)$  response function might lead to experimental methods of estimating the penetration depth of the radiant energy for particular samples.

*Keywords: Heat transfer; radiation; penetration; back-face rise time; midplane; front surface; V(t) response function; experimental methods; penetration depth; radiant energy; particular samples.*

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## PREFACE

The author is grateful for helpful discussions with Lou R. McCreight, Dr. Gary Hawkins, Dr. James Gee, and Dr. Tom Bahder of The Aerospace Corporation; Dr. Walt Eatherly of Oak Ridge National Laboratories, and Prof. T. W. Tong of the University of Kentucky.

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## I. INTRODUCTION

Thermal insulation characterization of polymers is an important requirement for present and future aerospace systems. Examples include a Neoprene/Kevlar composite used in rocket motors, as well as insulating blankets of fused silica or other materials. Furthermore, the construction of new homes requires the use of lightweight fibrous insulation whose thermal properties need to be known.

The significance of thermal radiation in fibrous insulations has been demonstrated<sup>1,2</sup> by conducting guarded hot-plate experiments under vacuum. Tong and Tien<sup>3,4</sup> categorized the analyses of such experiments as those which experimentally determine effective conductivities caused by radiation and those which consider the detailed equations governing the intensity of radiation in an absorbing and scattering medium.

The thermal diffusivity  $\alpha$  is the parameter of interest in transient-heat conduction problems, as opposed to steady-state problems in which only the thermal conductivity  $K$  plays a role. For a homogeneous body,  $\alpha$  may be calculated from  $K$ , the density  $\rho$ , and the specific heat  $c$ . If  $\alpha$ ,  $\rho$ , and  $c$  are measured in a given temperature range,  $K$  may be calculated over that range. This is especially advantageous when steady-state calorimetry for  $K$  is difficult, e.g. at high temperatures.

One method of measuring the thermal diffusivity is the flash technique<sup>5-12</sup>. A flash of radiant energy is deposited over one face of a disk-shaped sample that is surrounded by a heating element and enclosed in a vacuum. For a homogeneous sample,  $\alpha$  may be calculated from the sample thickness  $L$  and the time  $t_{1/2}$  (the time for the rear face to achieve one half its ultimate temperature). To within a numerical factor,  $\alpha$  is essentially  $L^2/t_{1/2}$ .

It has been observed<sup>12</sup> that problems involving large temperature differences and short heat-pulse propagation times may lead to situations in which the basic equations, boundary conditions, and assumptions of the flash

technique data-reduction scheme are inapplicable. For these problems there may be no way to convert an observed oscilloscope trace into a meaningful value for some physical parameter. Conditions occurring during space vehicle reentry, laser irradiation problems, measurements at cryogenic temperatures, and materials with anisotropic conductivity may lead to situations in which the flash diffusivity method may not be viable.

On the other hand, some nonhomogeneous materials have been successfully characterized by the flash technique. The flash technique has been applied to some heterogeneously dispersed composites and layered samples, including some carbon fibers, fiber-reinforced materials, individual layers of layered composites, and dispersed composites.<sup>12</sup>

The continuing challenge of composite-sample characterization by means of the flash technique is illustrated by recent case histories and journal publications. Flash-technique analysis<sup>13</sup> of a Neoprene-covered Kevlar cloth composite revealed that the diffusivity value could not be calculated for the Kevlar layer solely on the basis of the measured rise times and the properties of the virgin Neoprene. The rise times were too short, indicating that the conductivity values for the virgin Neoprene were too small for the Neoprene in the composite. In a second example, fibrous insulation, it has been shown that steady-state and transient methods of determining thermal conductivity often give different results, especially for such porous insulating materials as rockwool and kapok.<sup>2</sup> This discrepancy arises from the contribution of internal radiation to the conduction of heat and should be even larger for an intrinsically transparent material. Although recent publications<sup>3,4</sup> have addressed this issue from other viewpoints, internal radiation has not, to the present author's knowledge, previously been allowed for in the development of the flash technique.

Therefore, we see that some currently interesting sample types are in some way too complicated for available flash-technique analysis. The instrumentation is producing experimental outputs that are apparently in need of more refined interpretation. In any particular case, any one of a number

of different adjustments may be necessary. These adjustments are necessitated by four categories of physical effects:

1. Radiant-energy penetration during flash heating of the diffusivity sample (time = 0).
2. The contribution of internal radiation and absorption to heat transfer (time > 0).
3. Heterogeneous sample media.
4. Anisotropic sample media.

An adjustment accounting for the first effect and applying to both the layered-composite and porous-insulation types is discussed in this paper. This mathematical modification of the current flash-technique data-reduction method explicitly allows for an exponentially decaying spatial penetration of incident radiant energy into a layered sample. A method of experimentally evaluating the characteristic length associated with the exponential penetration function will be proposed.

## II. THEORETICAL DEFINITION OF THE PROBLEM

### A. HOMOGENEOUS SLAB

For a homogeneous slab between  $x = 0$  and  $x = L$ , the governing equations (as in Ref. 14) are

$$\vec{J} = -K \vec{\nabla} T \quad (1)$$

$$\vec{\nabla} \cdot \vec{J} + \rho c \frac{\partial T}{\partial t} = 0 \quad (2)$$

where  $T = T(x,t)$  is the temperature as a function of position  $x$  and time  $t$ ,  $K$ ,  $\rho$ , and  $c$  are the thermal conductivity, density, and specific heat, respectively, and  $J$  is the flux (energy per area and time). Homogeneity implies constant  $K$ ,  $\rho$ , and  $c$ , so combining the equations gives the partial differential equation (PDE)

$$\alpha \nabla^2 T = \frac{\partial T}{\partial t} \quad (3)$$

where

$$\alpha = \frac{K}{\rho c} \quad (4)$$

The boundary conditions (BC) for the homogeneous slab are  $\vec{\nabla} T = 0$  at  $x = 0$  and  $L$  (no heat loss). The initial conditions (IC) are

$$T(x,0) = \frac{Q}{\rho c} \delta(x) \quad (5)$$

where  $Q$  is the energy per area deposited on the slab surface ( $x = 0$  at  $t = 0$ ), and  $\delta(x)$  is the Dirac delta function. Separation of variables [ $T(x,t) \equiv W(x)\tau(t)$ ] leads to the ordinary differential equations (ODE)  $W'' + (\lambda/\alpha)W = 0$  and  $\tau' + \lambda\tau = 0$ , where  $\lambda$  is the separation constant and primes denote differentiation. In general for this Sturm-Liouville problem,

$W_n = A_n \sin \beta_n x + B_n \cos \beta_n x$  and  $\tau = e^{-\lambda t} = e^{-\beta_n^2 \alpha t}$ . The above BC imply that all  $A_n = 0$ , and the eigenvalue equation  $\sin \beta_n L = 0$  leads to

$$\beta_n = \frac{n\pi}{L}, \quad n = 0, 1, 2, \dots \quad (6)$$

(including negative  $n$  only changes the  $B_n$  definition). The  $B_n$  are specified by the IC with the result

$$T(x, t) = \frac{Q}{\rho c L} \left[ 1 + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi x}{L} \exp \left( -\frac{n^2 \pi^2}{L^2} \alpha t \right) \right] \quad (7)$$

The ultimate  $x = L$  temperature is  $Q/\rho c L$ , so the dimensionless temperature versus time profile at  $x = L$  is

$$V = 1 + 2 \sum_{n=1}^{\infty} (-1)^n \exp \left( -\frac{n^2 \pi^2}{L^2} \alpha t \right) \quad (8)$$

Approximating the  $V$  result by the first two terms (valid as  $t \rightarrow \infty$ ), one observes that  $V = 1/2$  when

$$\alpha = \frac{L^2 \ln 4}{\pi^2 t_{1/2}} = 0.1404 \frac{L^2}{t_{1/2}} \quad (9)$$

Using the full series expression,<sup>12</sup>

$$\alpha = 0.1388 \frac{L^2}{t_{1/2}} \quad (10)$$

Thus, the thermal diffusivity of a homogeneous slab can be measured by measuring the thickness  $L$  and the rise time  $t_{1/2}$ . This can be done to within 0.5% precision.<sup>12</sup> On the other hand, the IC and BC assumptions, such as delta function temperature-pulse across the entire sample and complete thermal isolation, limit the accuracy of the measurement. Models have been developed that generate corrections up to 21% in magnitude in the measured values.<sup>12</sup>

## B. COMPOSITE SLAB

Surprisingly enough, although a unique solution can also be shown to exist for a composite slab consisting of individual layers, that solution cannot be constructed by an extension of the Sturm-Liouville approach (p. 414 of Ref. 15). Instead, the Laplace transform technique (Chapter XII in Ref. 14) is used. The standard analysis for the simplest composite (two layers and one-dimensional heat flow) will first be summarized, using mostly the notation of Ref. 10. Then, in the following sections, recommendations for modifying the mathematics and the measurement algorithm will be stated, relying on the standard analysis as a starting point.

The two-layer problem is defined by the following equations:

Partial differential equations (PDE):

$$K_1 \nabla^2 T_1 = \rho_1 c_1 \frac{\partial T_1}{\partial t} \quad -\ell_1 < x < 0 \quad (11)$$

$$K_2 \nabla^2 T_2 = \rho_2 c_2 \frac{\partial T_2}{\partial t} \quad 0 < x < \ell_2 \quad (12)$$

Initial conditions (IC):

$$T_1(x, 0) = 0 \quad -\ell_1 < x < 0 \quad (13)$$

$$T_2(x, 0) = 0 \quad 0 < x < \ell_2 \quad (14)$$

Boundary conditions (BC):

$$-K_1 \nabla T_1(-\ell_1, t) = Q\delta(t) \quad (\text{flux pulse}) \quad (15)$$

$$\nabla T_2(\ell_2, t) = 0 \quad (\text{no back-side heat loss}) \quad (16)$$

$$T_1(0, t) = T_2(0, t) \quad (\text{no interfacial resistance}) \quad (17)$$

$$K_1 \nabla T_1(0, t) = K_2 \nabla T_2(0, t) \quad (\text{zero divergence at interface}) \quad (18)$$

Notice particularly that the temperature pulse in the IC of the Sturm-Liouville problem has been replaced by a flux pulse in the BC of this Laplace problem. Combining the above equations, we have the following results:

PDE & IC:

$$K_1 \frac{d^2 \bar{T}_1}{dx^2} - \rho_1 c_1 \{s \bar{T}_1 - 0\} = 0 \quad (19)$$

$$K_2 \frac{d^2 \bar{T}_2}{dx^2} - \rho_2 c_2 \{s \bar{T}_2 - 0\} = 0 \quad (20)$$

ODE:

$$\frac{d^2 \bar{T}_1}{dx^2} - \frac{s}{\alpha_1} \bar{T}_1 = 0 \quad (21)$$

$$\frac{d^2 \bar{T}_2}{dx^2} - \frac{s}{\alpha_2} \bar{T}_2 = 0 \quad (22)$$

BC:

$$-K_1 \left( \frac{d \bar{T}_1}{dx} \right)_{x = -\ell_1} = Q \quad s > 0 \quad (23)$$

$$\left( \frac{d \bar{T}_2}{dx} \right)_{x = \ell_2} = 0 \quad s > 0 \quad (24)$$

$$\bar{T}_1(0, s) = \bar{T}_2(0, s) \quad s > 0 \quad (25)$$

$$K_1 \frac{d \bar{T}_1}{dx} = K_2 \frac{d \bar{T}_2}{dx} \quad x = 0, s > 0 \quad (26)$$

Letting  $\omega_1 \equiv \sqrt{s/\alpha_1}$ , the ODEs in Eqs. (21) and (22) have the solutions  $\bar{T}_1 = A_1 \cosh \omega_1 x + B_1 \sinh \omega_1 x$ . Using the BCs in Eqs. (23) through (26), one obtains



$$\underline{a} \begin{pmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ p \end{pmatrix} \quad (27)$$

where the matrix  $\underline{a}$  has the elements  $a_{ij}$  ( $i, j = 1, 2, 3, 4$ ), and  $a_{13} = -\sinh \omega_2 \ell_2$ ,  $a_{14} = -\cosh \omega_2 \ell_2$ ,  $a_{21} = 1$ ,  $a_{23} = -1$ ,  $a_{32} = K_1 \omega_1$ ,  $a_{34} = -K_2 \omega_2$ ,  $a_{41} = \sinh \omega_1 \ell_1$ ,  $a_{42} = -\cosh \omega_1 \ell_1$ , all other  $a_{ij}$  being zero. Furthermore,  $p = Q/K_1 \omega_1$ .

Solving by determinants,

$$\det(\underline{a}) = K_2 \omega_2 \cosh \omega_1 \ell_1 \sinh \omega_2 \ell_2 + K_1 \omega_1 \sinh \omega_1 \ell_1 \cosh \omega_2 \ell_2 \quad (28)$$

$$A_1 = p K_1 \omega_1 \cosh \omega_2 \ell_2 / \det(\underline{a}) \quad (29)$$

$$B_1 = -p K_2 \omega_2 \sinh \omega_2 \ell_2 / \det(\underline{a}) \quad (30)$$

$$A_2 = p K_1 \omega_1 \cosh \omega_2 \ell_2 / \det(\underline{a}) \quad (31)$$

$$B_2 = -p K_1 \omega_1 \sinh \omega_2 \ell_2 / \det(\underline{a}) \quad (32)$$

Introducing

$$\sigma \equiv \frac{K_2 \omega_2}{K_1 \omega_1} = \sqrt{\frac{K_2 \rho_2 c_2}{K_1 \rho_1 c_1}} \quad \text{and} \quad \sqrt{U_1 s} \equiv \ell_1 \omega_1$$

and noting that  $\omega_1 K_1 = \sqrt{s} \sqrt{K_1 \rho_1 c_1}$ , Eqs. (28) through (32) become

$$\det(\underline{a}) = \sqrt{s} \sqrt{K_1 \rho_1 c_1} [\sinh \sqrt{U_1 s} \cosh \sqrt{U_2 s} + \sigma \cosh \sqrt{U_1 s} \sinh \sqrt{U_2 s}] \quad (33)$$

$$A_1 = Q \cosh \omega_2 l_2 / \det (\underline{a}) \quad (34)$$

$$B_1 = Q (-\sigma \sinh \omega_2 l_2) / \det (\underline{a}) \quad (35)$$

$$A_2 = Q \cosh \omega_2 l_2 / \det (\underline{a}) \quad (36)$$

$$B_2 = Q (-\sinh \omega_2 l_2) / \det (\underline{a}) \quad (37)$$

Hence

$$\bar{T}_1 = \frac{Q}{\det (\underline{a})} (\cosh \omega_2 l_2 \cosh \omega_1 x - \sigma \sinh \omega_2 l_2 \sinh \omega_1 x) \quad (38)$$

$$\bar{T}_2 = \frac{Q}{\det (\underline{a})} (\cosh \omega_2 l_2 \cosh \omega_2 x - \sinh \omega_2 l_2 \sinh \omega_2 x) \quad (39a)$$

$$\bar{T}_2 = \frac{Q}{\det (\underline{a})} \cosh [\sqrt{U_2} s (1 - x/l_2)] \quad (39b)$$

Combining Eqs. (33) and (39b) and introducing  $T_0 \equiv \frac{Q}{\sqrt{K_1 \rho_1 c_1}}$ , we have

$$\frac{\bar{T}_2}{T_0} = \frac{1}{s^{1/2}} \frac{\cosh [\sqrt{U_2} s (1 - x/l_2)]}{\sinh \sqrt{U_1} s \cosh \sqrt{U_2} s + \sigma \cosh \sqrt{U_1} s \sinh \sqrt{U_2} s} \quad (40)$$

On the entire complex plane the series expansions of the factors in the denominator vary as  $\sqrt{s} [\sqrt{s} (1 + \text{even powers})] (1 + \text{even powers})$ . Therefore the denominator behaves as  $s$  near  $s = 0$  (simple pole at  $s = 0$ ). The nature of the remaining roots of the denominator has been investigated (pp. 324-326 of Ref. 14, pp. 409-410 of Ref. 15 and Appendix A of Ref 16). There are no complex roots. The function  $\bar{T}_2/T_0$  is meromorphic, with poles at  $s = 0$  and  $s = s_n$ ,  $n = 1, 2, 3, \dots$ .

The Mittag-Leffler Theorem states that any meromorphic function can be expressed as a sum of an entire function and a series of rational functions. In our case, letting  $f(s) = \bar{T}_2/T_0$

$$f(s) = \sum_{n=0}^{\infty} \frac{r_n}{s - s_n} + \frac{1}{2\pi i} \int_c \frac{f(s') ds'}{s' - s} \quad (41a)$$

where  $r_n$  is the residue of the  $n^{\text{th}}$ /pole. The contour integral in Eq. (41a) can be shown to be zero as in standard texts on complex analysis. Thus

$$f(s) = \frac{r_0}{s} + \sum_{n=1}^{\infty} \frac{r_n}{s - s_n} \quad (41b)$$

$$f(s) = \frac{1}{s} \lim_{s \rightarrow 0} s f(s) + \sum_{n=1}^{\infty} \frac{X(s_n)}{Y'(s_n)(s - s_n)} \quad (41c)$$

where

$$X(s) = \frac{\cosh [\sqrt{U_2} s (1 - x/l_2)]}{\sqrt{s}} \quad (42)$$

$$Y(s) = \sinh \sqrt{U_1} s \cosh \sqrt{U_2} s + \sigma \cosh \sqrt{U_1} s \sinh \sqrt{U_2} s \quad (43)$$

Thus, letting  $X = \sqrt{U_1/U_2}$ , we obtain

$$\frac{\bar{T}_2}{T_0} = \frac{1}{s \sqrt{U_2} (X + \sigma)} + \sum_{n=1}^{\infty} \frac{X(s_n)}{Y'(s_n)(s - s_n)} \quad (44)$$

Using the inverse Laplace operator,  $\mathcal{L}^{-1}$ , one obtains

$$\frac{T_2}{T_0} = \frac{1}{\sqrt{U_2} (X + \sigma)} + \sum_{n=1}^{\infty} \frac{X(s_n)}{Y'(s_n)} e^{s_n t} \quad (45)$$

Consider the following definitions:

$$\beta_n = \sqrt{-U_2 s_n} \quad n = 1, 2, 3, \dots \quad (46a)$$

$$\Omega = \frac{X\sigma + 1}{X + 1} \quad (46b)$$

$$\frac{1}{\sigma} = \frac{H}{X} \quad (46c)$$

$$U_i = \frac{\ell_i^2}{\alpha_i}, \quad i = 1, 2 \quad (46d)$$

After considerable algebra, it results that

$$T_2 = \frac{Q}{\ell_1 \rho_1 c_1 + \ell_2 \rho_2 c_2} \left\{ 1 + 2 \sum_{n=1}^{\infty} \frac{e^{-\beta_n^2 t / U_2} \cos [\beta_n (1 - x / \ell_2)]}{\cos X \beta_n \cos \beta_n - \Omega \sin X \beta_n \sin \beta_n} \right\} \quad (47a)$$

$$V(t) = \frac{T_2(\ell_2, t)}{T_{2, \max}} \quad (47b)$$

where  $T_{2, \max} = Q / [\ell_1 \rho_1 c_1 + \ell_2 \rho_2 c_2]$ . The equation that gives the roots of the denominator in Eq. (40), when recast in terms of circular functions, determines the real, positive eigenvalues  $\beta = \beta_n = \sqrt{-U_2 s_n}$  according to

$$\sin X \beta \cos \beta + \sigma \cos X \beta \sin \beta = 0 \quad (47c)$$

Furthermore, when the slabs have identical parameters,

$$T_2 = \frac{Q}{2\ell_1 \rho_1 c_1} \left\{ 1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2 \pi^2 \alpha_1 t}{4\ell_1^2}} \cos \left[ \frac{n\pi}{2} (1 - x / \ell_2) \right] \right\} \quad (47d)$$

which corresponds to the  $V$  result given in the homogeneous sample analysis. This shows that the flux-pulse, dual-slab, Laplace analysis can reduce to the temperature-pulse, homogeneous-slab, Sturm-Liouville analysis for an

appropriate choice of parameters. Finally, if  $V(t)$  or  $\tau_{1/2}$  is measured and compared to Eqs. (47a) or (47b), one unknown material parameter can be obtained by standard computer techniques.

### III. ANALYSIS OF THE RADIANT-ENERGY PENETRATION EFFECT

Suppose that the incident electromagnetic energy is converted to heat throughout the depth of the first layer of a two-layer composite according to

$$Q(x) = \frac{Qk e^{-k(x + \ell_1)}}{(1 - e^{-k\ell_1})}, \quad -\ell_1 < x < 0 \quad (48)$$

The normalization was chosen to ensure that  $Q$  energy units per area are incident. This is the same  $Q$  as in the standard solution above, in which all the electromagnetic energy is converted to heat at the front interface. We will first analyze what effect this penetration will have on the observable quantity  $t_{1/2}$ .

Equations (11) through (18) are the same as before, with the following exceptions:

$$T_1(x, 0) = \frac{Qk e^{-k(x + \ell_1)}}{\rho_1 c_1 (1 - e^{-k\ell_1})}, \quad -\ell_1 < x < 0 \quad (13')$$

and

$$-K_1 \nabla T_1(-\ell_1, t) = 0 \quad (15')$$

Now Eqs. (19) through (26) are the same as before, with the following exceptions:

$$K_1 \frac{d^2 \bar{T}_1}{dx^2} - \rho_1 c_1 \left\{ s \bar{T}_1 - \frac{Qk e^{-k(x + \ell_1)}}{\rho_1 c_1 (1 - e^{-k\ell_1})} \right\} = 0 \quad (19')$$

and

$$-\left(K_1 \frac{d\bar{T}_1}{dx}\right)_{x=-\ell_1} = 0 \quad (23')$$

The solution to the ODE, Eq. (19'), is somewhat different than before:

$$\bar{T}_1(x,s) = A_1 \cosh \omega_1 x + B_1 \sinh \omega_1 x + \frac{Qke^{-k(x+\ell_1)}}{K_1(1-e^{-k\ell_1})(\omega_1^2 - k^2)} \quad (19'')$$

Now the matrix equation, Eq. (27) becomes

$$\underline{a} = \begin{pmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 \\ p'' \\ p' \\ p \end{pmatrix} \quad (27')$$

All the entries of  $\underline{a}$  are the same, but now

$$p'' = \frac{-Qke^{-k\ell_1}}{K_1(1-e^{-k\ell_1})(\omega_1^2 - k^2)} \quad (49a)$$

$$p' = \frac{+Qke^{-k\ell_1}}{(1-e^{-k\ell_1})(\omega_1^2 - k^2)} \quad (49b)$$

$$p = \frac{-Qk^2}{\omega_1 K_1(1-e^{-k\ell_1})(\omega_1^2 - k^2)} \quad (49c)$$

We will assume only a limited penetration consistent with  $e^{-k\ell_1} \lesssim 0.05$ , or  $k\ell_1 \gtrsim 3$ .

To a reasonable approximation, therefore, we set  $e^{-k\ell_1}$ ,  $p''$ , and  $p'$  equal to zero, and

$$p = \frac{Q}{K_1 \omega_1 \left(1 - \frac{s}{k^2 \alpha_1}\right)} \quad (50)$$

In the limit that  $k \rightarrow \infty$  (delta function, or surface, energy deposition),  $p \rightarrow Q/K_1 \omega_1$  as in the standard solution above. Therefore, we can follow the changing solution as surface deposition is continuously changed to deposition in depth. We can do this rigorously if nonzero  $p''$  and  $p'$  are considered. Here we consider only the case of small penetration,  $k\ell_1 \gtrsim 3$ .

Since the problem has only changed to the extent of a modification of the factor  $Q$  in the quantity  $p$ , it is seen immediately that

$$\frac{\bar{T}_2}{\bar{T}_0} = \frac{1}{s^{1/2}(1 - s/k^2 \alpha_1)} \frac{\cosh [\sqrt{U_2} s (1 - x/\ell_2)]}{[\sinh \sqrt{U_1} s \cosh \sqrt{U_2} s + \sigma \cosh \sqrt{U_1} s \sinh \sqrt{U_2} s]} \quad (40')$$

Hence the change in the problem which is due to finite radiant-energy penetration is seen in a change in the pole structure, as shown in Fig. 1.

Thus the previous solution, Eq. (41b), is modified to become

$$f(s) = \frac{r_0}{s} + \sum_{n=1}^{\infty} \frac{r_n}{s - s_n} + \frac{r_k}{s - s_k} \quad (41b')$$

Since  $k\ell_1 \gtrsim 3$ ,  $k$  is positive, and  $s_k > 0$ . Now, considering Eqs. (42), (43), and (40'),

$$r_0 = \frac{1}{\sqrt{U_2} (X + \sigma)} \quad (51a)$$



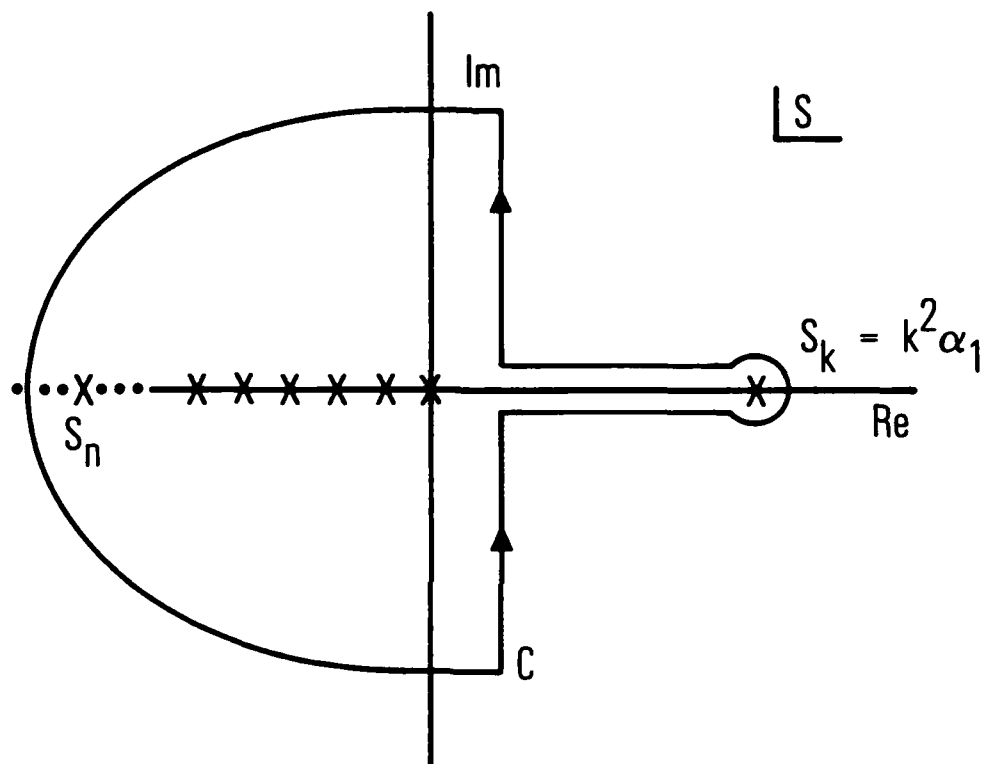


Fig. 1. The Pole Structure for the Two-Layer Composite with a Finite Penetration Depth of Electromagnetic Energy. The difference in this solution, compared to the case of electromagnetic energy conversion to heat at the surface only, is the appearance of a new pole at  $s_k = k^2 \alpha_1$ . C is the contour for the integration that inverts  $\bar{T}_2(x_1 s)/T_0$ .

$$r_n = \frac{-s_k X(s_n)}{(s_n - s_k) Y'(s_n)} \quad (51b)$$

$$r_k = -s_k \frac{X(s_k)}{Y(s_k)} \quad (51c)$$

We note that  $r_k$  is proportional to  $e^{-k\ell_1} \rightarrow 0$ , since we have taken  $e^{-k\ell_1} = 0$  as an approximation. Thus the last term in Eq. (41b') must be ignored. Only if  $p'' \neq 0$ ,  $p' \neq 0$ , and  $e^{-k\ell_1} \neq 0$  in Eq. (27') would it be consistent to keep such terms. In our approximation, then, Eq. (41b') differs from Eq. (41b) only through the  $-s_k/(s_n - s_k)$  multiplicative factor in Eq. (51b).

Using  $s_k = k^2 \alpha_1$  and  $s_n = -\beta_n^2/U_2$  Eq. (47a) becomes

$$\frac{T_2}{T_{2,\max}} = 1 + 2 \sum_{n=1}^{\infty} \frac{1}{1 + \beta_n^2/U_2 k^2 \alpha_1} \frac{e^{-\beta_n^2 t/U_2} \cos[\beta_n(1 - x/\ell_2)]}{\cos X \beta_n \cos \beta_n - \Omega \sin X \beta_n \sin \beta_n} \quad (52)$$

Equation (52) is the master equation for the  $p'' = p' = 0$  approximation to the finite-energy penetration problem. Two tasks now remain: First it will be demonstrated that this energy penetration can significantly affect experimental results. Then it will be indicated how the computer data-reduction algorithm might be modified to include codetermination of  $k$  along with the usual parameter evaluation.

The significance of Eq. (52) can be judged by evaluating it for the case of identical-layer parameters and in the first-term ( $n = 1$ ) approximation analogous to Eq. (9):

$$\frac{1}{2} = 1 - \frac{2 e^{-\pi^2 \alpha_1 t_{1/2}/4\ell_1^2}}{1 + \pi^2/4k^2 \ell_1^2} \quad (53a)$$

$$\frac{4\ell_1^2}{\pi^2 \alpha_1} \left[ \ln 4 - \ln \left( 1 + \frac{\pi^2}{4k^2 \ell_1^2} \right) \right] = t_{1/2} \quad (53b)$$

Letting  $t_{1/2}^{(0)}$  be the rise time for the spatial delta-function energy deposition [Eq. (9)], we have

$$t_{1/2} = t_{1/2}^{(0)} - \frac{4\ell_1^2}{\pi^2 \alpha_1} \ln\left(1 + \frac{\pi^2}{4k^2 \ell_1^2}\right) \quad (53c)$$

Letting SF be the fractional deviation (shortfall) of the rise time in the finite-deposition case, we obtain

$$SF = \frac{t_{1/2} - t_{1/2}^{(0)}}{t_{1/2}^{(0)}} = - \frac{\ln(1 + \pi^2/4k^2 \ell_1^2)}{\ln 4} \quad (53c)$$

for the approximation  $k\ell_1 \gtrsim 3$ . Figure 2 shows the result for the shortfall of the rise time versus  $k\ell_1$  or the characteristic penetration depth  $1/k$ .

The approximations used in the construction of Fig. 2 could be removed by the rigorous ( $p'' \neq 0$ ,  $p' \neq 0$ ,  $e^{-k\ell_1} \neq 0$ ) solution of Eq. (27'), the subsequent inclusion of the  $r_k$  residue in Eq. (41b'), and the use of the entire series in Eq. (52). However, even with those refinements the -SF ( $k\ell_1 = 3$ ) is still expected to be in the 10 to 20% range, even though the energy reaching the midplane of the sample is only  $e^{-3} = 5\%$  of that converted to heat at the front interface.

We see that the finite penetration of radiation may cause a significant rise-time shortfall. How can this effect be allowed for in data reduction?

In one standard form (p. 493 of Ref. 10) of the flash-technique data reduction, the eigenvalue equation [Eq. (47c)] is solved with the measured auxiliary parameters and a first guess at  $\alpha_1$ , which is chosen as the unknown. The first guess is  $\alpha_1 = 0.1388 \ell_1^2/t_{1/2}$ , as in Eq. 10. (Presumably  $\alpha_1 = 0.1404 \ell_1^2/t_{1/2}$  [Eq. (9)] would also lead to a convergent series.) The deviation from  $V(t = t_{1/2})$  is calculated, and an iteration algorithm is devised which terminates when  $\alpha_1^{i+1} - \alpha_1^i = \frac{1}{2}(\alpha_1^i + \alpha_1^{i-1})$  converges to a reasonable limit.

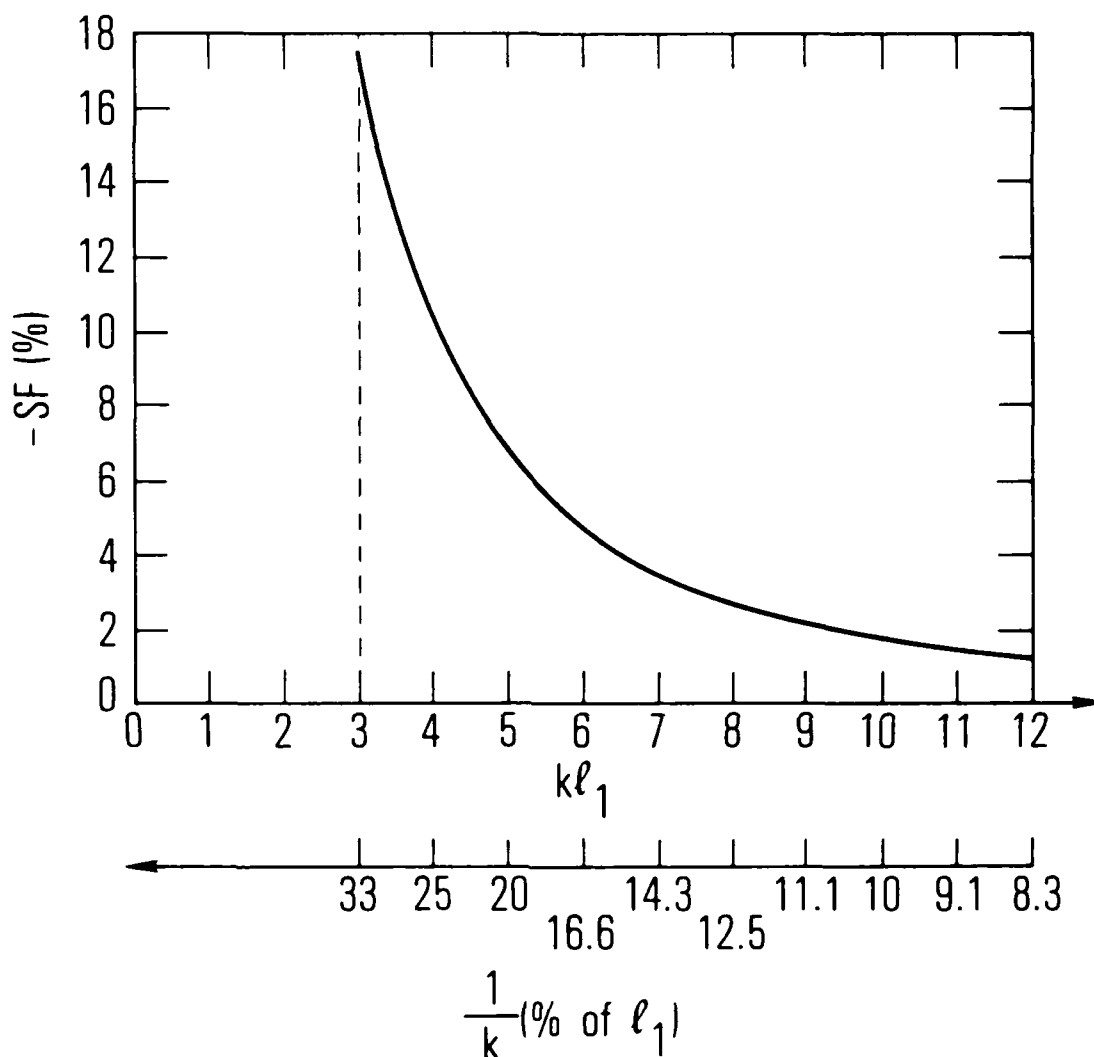


Fig. 2. The Magnitude of the Percent Shortfall ( $\sim$ SF) of the Measured Rise Time in the Thermal Diffusivity Measurement of a Homogeneous Layer of Width  $2\ell_1$ , Graphed as a Function of  $k\ell_1$  and of the Characteristic Penetration Depth of  $1/k$ . In our approximation ( $p'' = p' = e^{-k\ell_1} = 0$ ),  $k\ell_1 \gtrsim 3$ , and only the  $n = 1$  rise times are compared.

Bulmer and Taylor use only the  $t_{1/2}$  experimental output, because they feel that curve fitting to more points requires using extremes of the experimental curve which are inherently less accurate.

We would argue that the entire experimental  $V(t)$  curve contains some relevant information, and that at the very least one additional parameter,  $k$ , should be derivable from this information. We propose that a figure of merit,  $\text{RMS}^2(k \rightarrow \infty, \alpha_1^{i+1})$ , be formed for the first determination,  $\alpha_1^{i+1}$ , in the following manner:

$$\text{RMS}^2(k \rightarrow \infty, \alpha_1^{i+1}) = \frac{1}{N} \sum_{n=1}^N |V_{\text{exp}}(t_n) - V_k(t_n)|^2 \quad (54)$$

where  $N$  points  $t_n$  on the time scale are chosen.  $V_{\text{exp}}(t)$  is the experimental trace, and  $V_k(t)$  is given in Eq. (52), in this case with  $k \rightarrow \infty$ . Now a different initial guess can be made for  $\alpha_1$  using a finite value of  $k$ :

$$\alpha_1 = \frac{4\ell_1^2}{\pi^2 t_{1/2}} \left[ \ln 4 - \ln \left( 1 + \frac{\pi^2}{4k^2 \ell_1^2} \right) \right] \quad (55)$$

After convergence to a new  $\alpha_1^{i'+1}$ , the quantity  $\text{RMS}^2(k, \alpha_1^{i'+1})$  can be calculated. When  $\text{RMS}^2(k, \alpha_1^{i'+1})$  reaches a minimum as a function of  $k$ , that is the best  $k$  for the measurement.

#### IV. SUMMARY AND CONCLUSIONS

Current and future aerospace and construction industry materials require efficient thermal analysis techniques, including the flash technique for thermal diffusivity. This is not possible unless the data can be adequately reduced for polymer composites and porous blankets. This paper has been concerned with radiant-energy penetration into a layered sample subjected to flash-technique conditions. A mathematical modification of the standard flash-technique solution for layered structures was presented which allows for an exponentially decaying penetration of radiant energy into the first of two layers. The following conclusions were drawn:

- (1) A new (simple) pole in the Laplace-transformed solution appears on the complex  $s$  or  $\beta$  plane. Thus the inverse Laplace transform, which gives the response function (back-face temperature versus time), contains an additional term. Furthermore, there is a small modification  $(1 + \beta_n^2/U_2 k^2 \alpha_1)^{-1}$  to the previous terms.
- (2) The appearance of a new pole signifies in general the emergence of a new kind of physical behavior, because the residue of the new pole may in general be significantly larger than those of many of the original poles. However, to facilitate the evaluation of the importance of this effect, a small-penetration approximation was made in which  $e^{-k\ell_1}$  was set equal to zero.

This is approximately true for  $k\ell_1 \geq 3$  ( $e^{-k\ell_1} \leq 0.05$ ). In this approximation, the new term is suppressed and only the  $(1 + \beta_n^2/U_2 k^2 \alpha_1)^{-1}$  modification retained. However, it must be kept in mind that for more extensive radiant energy penetration, this approximation would not apply, in which case the new term in the solution could specify a radical departure of the response function from its previous behavior. Furthermore, extensive penetration might also require the exponential function  $Q(x)$  to extend over more than one layer.

- (3) A numerical example was evaluated for the  $e^{-k\ell_1} = 0$  approximation in the case of identical layer parameters and the first-term ( $n = 1$ ) approximation to the back-face rise time. It was found that the rise time shortfall ( $1/k = 33\%$  of  $\ell_1$ )  $\approx 18\%$  of the rise time that would obtain if all the radiant energy were deposited on the surface. Here,  $1/k$  is the characteristic penetration depth of the exponential radiant-energy deposition function. Thus, even when the radiant energy deposited at the sample midplane is only 5% of that deposited on the surface, an 18% correction is necessary.

- (4) The standard flash-technique data-reduction algorithm considers iterations,  $\alpha_1^{(i+1)} = \frac{1}{2} [\alpha_1^{(i)} + \alpha_1^{(i-1)}]$ , which are required to converge to a reasonable limit. Only the  $t_{1/2}$  experimental data are used. If the entire  $V(t)$  response function were used, however, a figure of merit might be defined that allows for the experimental determination of  $k$ .
- (5) Modifications such as the ones reported here are expected to open up the flash technique to the analysis of advanced polymer composite and insulating blanket material.

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# LIST OF SYMBOLS

$A_i, B_i$	constant coefficients in the solution of the ordinary differential equation for $\bar{T}_2$ .
$a$	matrix of the coefficients $A_i, B_i$ arising from the boundary conditions [see Eq. (27)]
$c$	specific heat
$f(s)$	$= \bar{T}_2(x,s)/T_0$
$H$	$= X/\sigma$
$i$	layer index for the composite slab (layer 1 is hit by the incident flash)
$J$	thermal flux
$K$	thermal conductivity
$1/k$	characteristic penetration depth of the radiant energy into the first layer of a composite slab ( $k \rightarrow \infty$ for surface deposition only)
$\mathcal{L}, \mathcal{L}^{-1}$	Laplace (inverse Laplace) operator
$L$	thickness of the (homogeneous) layer
$l_i$	thickness of the $i^{\text{th}}$ layer
$p, p', p''$	nonhomogeneous entries in Eqs. (17) or (17')
$Q$	heat per unit area deposited on sample
$r_0$	residue of $f(s)$ at $s = 0$
$r_n$	residue of $f(s)$ at $s = s_n$ ( $n = 1, 2, 3, \dots$ )
$r_k$	residue of $f(s)$ at $s = s_k$
$\text{RMS}^2(k, \alpha_1^{i+1})$	figure of merit for deviation of $V_k(t)$ from $V_{\text{exp}}(t)$ , using $\alpha_1 = \alpha_1$
$s$	Laplace-conjugate variable to the time $t$
$s_n$	location of the simple poles of $\bar{T}_2/T_0$ on the complex $s$ -plane ( $n = 1, 2, 3, \dots$ ) ( $s_0 = 0$ simple pole evaluated separately)

# LIST OF SYMBOLS (Continued)

$s_k$	$= k^2 \alpha_1$ = the new simple pole that appears as a result of finite radiant-energy penetration
SF	fractional shortfall of the measured rise time (penetration case) below that occurring for surface-energy deposition only [see Eq. (53c)]
$T_i(x, t)$	temperature in layer $i$ at position $x$ and time $t$
$\bar{T}_i(s, t)$	Laplace transform of $T_i$
$t_{1/2}$	characteristic rise time of the back-face temperature of the sample in the flash technique
$T_{2, \max}$	$= Q / [\ell_1 \rho_1 c_1 + \ell_2 \rho_2 c_2]$
$T_0$	$= Q / \sqrt{K_1 \rho_1 c_1}$
$t_{1/2}^{(0)}$	back-face rise time when there is surface-energy deposition only
$U_1$	$= \ell_1^2 / \alpha_1$
$\sqrt{U_1 s}$	$= \ell_1 \omega_1$
$V(t)$	back-face response function
$V_{\exp}(t)$	measured back-face response function
$V_k(t)$	back-face response function [see Eq. (52)] for the case of finite $k$
$X$	$= \sqrt{U_1 / U_2}$
$X(s), Y(s)$	useful functions for residue evaluation [see Eqs. (42) and (43)]
$\alpha$	thermal diffusivity
$\alpha_1^i$	the value of $\alpha_1$ after $i$ iterations of the standard data-reduction algorithm for the flash technique
$\beta_n$	eigenvalues of the boundary-condition problem ( $= \sqrt{-U_2 s_n}$ , $n = 1, 2, 3, \dots$ )
$\nabla$	differential operator

# LIST OF SYMBOLS (Continued)

$\delta$	Dirac delta function
$\rho$	density
$\sigma$	$= K_2 \omega_2 / K_1 \omega_1 = \sqrt{K_2 \rho_2 c_2 / K_1 \rho_1 c_1}$
$\Omega$	$(X\sigma + 1)/(X + 1)$
$\omega_1$	$= \sqrt{s/\alpha_1}$

## LABORATORY OPERATIONS

The Laboratory Operations of The Aerospace Corporation is conducting experimental and theoretical investigations necessary for the evaluation and application of scientific advances to new military space systems. Versatility and flexibility have been developed to a high degree by the laboratory personnel in dealing with the many problems encountered in the nation's rapidly developing space systems. Expertise in the latest scientific developments is vital to the accomplishment of tasks related to these problems. The laboratories that contribute to this research are:

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Chemistry and Physics Laboratory: Atmospheric chemical reactions, atmospheric optics, light scattering, state-specific chemical reactions and radiation transport in rocket plumes, applied laser spectroscopy, laser chemistry, laser optoelectronics, solar cell physics, battery electrochemistry, space vacuum and radiation effects on materials, lubrication and surface phenomena, thermionic emission, photosensitive materials and detectors, atomic frequency standards, and environmental chemistry.

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